Outline	Random variable and Observation	Random Sample 0 00	Statistic ○ ○	Sampling Distribution O O O	Exercises

# Chapter 6 - Lecture 1 Statistics and their distribution

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Chapter 6 - Lecture 1 Statistics and their distribution

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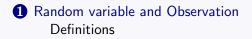
Outline	Random variable and Observation	<b>Random Sample</b> o oo	Statistic 0 0	Sampling Distribution o o o	Exercises

### Overview of Chapter 6

Chapter 6 : Statistics and Sampling Distributions

- **1** 6.1 Statistics and Their Distributions
  - Introduce definitions.
- **2** 6.3 The Distribution of a Linear Combination
  - Tools for proofs.
- **3** 6.2 The Distribution of the Sample Mean
  - Important properties for sample mean
- **4** 6.4 Distribution Based on a Normal Random Sample
  - Introduce several important distributions.

Outline	Random variable and Observation	Random Sample	Statistic	Sampling Distribution	Exercises
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**2** Random Sample

Definitions Examples

#### 3 Statistic

Definition Examples

#### **4** Sampling Distribution

Definition Finding Sampling Distributions Example



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Outline	Random variable and Observation ●○○	Random Sample 0 00	Statistic ○ ○	Sampling Distribution O O	Exercises
Definitions					

- Random variables  $X_1$  and  $X_2$ , ..., $X_n$ , where *n* denotes the **sample size**.
- Let say, I have random variables .... So if I get a random sample of size 3, *n* = 3.
- If in observation, X<sub>1</sub> takes value 1, then denote it as x<sub>1</sub> = 1.
  and let's say the values are x<sub>1</sub> = 2, x<sub>2</sub> = 1, x<sub>3</sub> = 1.
- What is the difference between  $X_1$  and  $x_1$ ?

Outline	Random variable and Observation ○●○	Random Sample 0 00	Statistic ○ ○	Sampling Distribution O O O	Exercises
Definitions					

### Random variable and Observation

- X denotes a random variable which is unknown.
- x denotes the **observed value** of the random variable which is known and might be different from sample to sample.

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Outline	Random variable and Observation ○○●	Random Sample 0 00	Statistic ○ ○	Sampling Distribution O O O	Exercises
Definitions	:				

### Random variable and Observation

- Random variables have an uncertainty for their values.
- That means two things:
  - You do not know what the value of random variables are until you actually see the observed values in the sample.
  - Any value depending on random variables will be expected to differ from sample to sample.

Outline	Random variable and Observation	Random Sample ● ○○	Statistic ○ ○	Sampling Distribution O O	Exercises
Definitions	3				

# Random sample = iid

- What a random sample is?
  - All random variables are independent
  - All random variables come from the same distribution (as from the population), that is they are identically distributed
- In short, we write **iid**, which means independent and identically distributed
- Intuitively, random sample is the sample that is representative of the population.

Outline	Random variable and Observation	Random Sample ● ○○	Statistic ○ ○	Sampling Distribution O O	Exercises
Definitions	3				

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Randomness of Sample is always tricky. In this course we just assume it unless otherwise explicitly specified.

Outline	Random variable and Observation	Random Sample ○ ●○	Statistic 0 0	Sampling Distribution O O	Exercises
Examples					

#### Do you know $\pi$ ?

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Outline	Random variable and Observation	Random Sample ⊙ ●○	Statistic ○ ○	Sampling Distribution O O O	Exercises
Examples					

#### Do you know $\pi$ ? How many digits you can tell?

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Outline	Random variable and Observation	Random Sample ⊙ ●○	Statistic ○ ○	Sampling Distribution O O O	Exercises
Examples					

Do you know  $\pi$  ? How many digits you can tell?

Required Readings: Are the Digits of  $\pi$  an Independent and Identically Distributed Sequence?

Outline	Random variable and Observation	Random Sample ○ ○●	Statistic 0 0	Sampling Distribution O O	Exercises
Examples					

Not example:

1 Convenient Sample: to select sample that is easy to get

- 1 Select your family members;
- **2** Select your friends and classmates;
- **3** Select people you know on Facebook and twitter;
- Oata snooping: Select the part of sample that you prefer, ignore the rest
  - Learn more about the data snooping: http://data-snooping.martinsewell.com/

Outline	Random variable and Observation	Random Sample o oo	Statistic ● ○	Sampling Distribution O O	Exercises
Definition					

Definition of Statistic

- We call **statistic** any quantity whose value can be calculated from sample data. That means a statistic is a function of random variables from our random sample  $X_1, \ldots, X_n$ .
- Do you think a statistic should be denoted with an upper case letter or a lower case letter?

Outline	Random variable and Observation	Random Sample o oo	Statistic O	Sampling Distribution o o o	Exercises
Examples					
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Examples

A statistic is also a random variable.

**1** Sample Mean: 
$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

**2** Sample Variance: 
$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

**3** Other examples: sample quantiles, sample standard deviation, etc.

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Outline	Random variable and Observation	Random Sample o oo	Statistic 0 0	Sampling Distribution • •	Exercises
Definition					
Sam	oling Distribution				

• The probability distribution of a statistic is called **sampling distribution** to emphasize the fact that it describes how the statistic varies from one random sample to another.

Outline	Random variable and Observation	Random Sample	Statistic	Sampling Distribution	Exercises
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Finding Sampling Distributions

How we find the sampling distribution of a statistic

- Using Probability Rules. (e.g 6.2)
- Simulation Experiments.
- Using known theorems (which is considered an extension of the first case). (section 6.3,6.4)

Outline	Random variable and Observation	Random Sample o oo	Statistic 0 0	Sampling Distribution	Exercises
Example					
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### Example 6.2 page 282

Example 1: Suppose  $(X_1, X_2)$  is a random sample of size 2 and each of them has the following probability distribution:

Table: Probability distribution of  $X_1(X_2)$ 

X	40	45	50
p(x)	0.2	0.3	0.5

- What is the probability distribution of  $\bar{X} = \frac{X_1 + X_2}{2}$ ?
- What is the probability distribution of S<sup>2</sup>?

Outline	Random variable and Observation	Random Sample 0 00	Statistic ○ ○	Sampling Distribution O O O	Exercises



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