Outline	General Properties of Sample Mean	For Normal Population (Exactly)	For general populations (asymptotically)	Ho
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Chapter 6 - Lecture 3 The distribution of the sample mean

Yuan Huang

January 14th, 2013

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1 General Properties of Sample Mean

Mean and Variance Asymptotic properties: Law of Large Numbers

2 For Normal Population (Exactly)

Distribution of Sample Mean Example

For general populations (asymptotically)
 Central Limit Theorem
 Examples
 Applications



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General Properties of Sample Mean

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General: Mean and variance of Sample Mean

A random sample $X_1, X_2, ..., X_n$ from a distribution with mean μ and variance σ^2 , then the distribution of the sample mean \bar{X} has:

1 mean
$$E(\bar{X}) = \mu_{\bar{X}} = \mu$$

2 variance $V(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$

It means:

Mean and Variance

- The sampling distribution of \bar{X} is centered precisely at the mean of the population from which the sample has been selected.
- 2 The sampling distribution of \bar{X} becomes more concentrated about μ as the sample size *n* increases.

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Mean and Variance

Law of Large Numbers

$$ar{X}
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?, as $n
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Law of Large Numbers (LLN)

Thm: If X_1, \ldots, X_n is a random sample from a distribution with mean μ and variance σ^2 , then as $n \to \infty$, \bar{X}_n converges to μ :

- In mean square $E[(ar{X}-\mu)^2]
 ightarrow 0$
- In probability $P(|ar{X}-\mu|\geq\epsilon)
 ightarrow 0$

Remarks: LLN is one theoretical support of using large sample size - as sample size goes large the sample estimate becomes accurate.

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Asymptotic properties: Law of Large Numbers					

For weak law:

Easy way to remember: The chance that \bar{X} is far away from μ is going to 0 as sample size is growing!

[Proof]: By Chebyshev's Inequality,

$$\begin{split} \mathsf{P}(|\bar{X} - \mu| \geq \epsilon) &= \mathsf{P}\left(|\bar{X} - \mu| \geq \left(\epsilon \frac{\sqrt{n}}{\sigma}\right) \frac{\sigma}{\sqrt{n}}\right) \\ &\leq \frac{1}{\left(\epsilon \frac{\sqrt{n}}{\sigma}\right)^2} = \frac{\sigma^2}{n\epsilon^2} \end{split}$$

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Distribu	tion of Sample Mean			

Distribution of Sample Mean for Normal distribution

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Distribution of Sample Mean

Distribution of Sample Mean for Normal distribution

A random sample $X_1, X_2, ..., X_n$ from a normal distribution with mean μ and variance $\sigma^2 (X_i \sim N(\mu, \sigma^2))$ then the sample mean \bar{X} has a sampling distribution which is :

- normally distributed (by prop 3 from Lec 6.2)
- with mean $\mu_{ar{X}}=\mu$ (by prop 1 from Lec 6.2)

• with variance
$$\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$$
 (by prop 2 from Lec 6.2)

In short as:
$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Outline General Properties of Sample Mean	For Normal Population (Exactly) ○○ ●	For general populations (asymptotically) 000 000	Но
Example			

Exercise 6.19

[6.19] Suppose the sediment density of a randomly selected specimen from a certain region is normally distributed with mean 2.65 and standard deviation 0.85. If a random sample of 25 specimen is selected, what is the probability that sample mean is at most 3.00 ?

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Asymptotic distribution of \bar{X} for general populations

Central Limit Theorem

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Central Limit Theorem (CLT)

Thm:. If X_1, \ldots, X_n is a random sample with mean μ and variance σ^2 , then as $n \to \infty$, the limiting distribution of $\sqrt{n}(\bar{X}_n - \mu)/\sigma$ is standard normal, written as

$$rac{\sqrt{n}(ar{X}_n-\mu)}{\sigma}
ightarrow_d N(0,1).$$

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Central Limit Theorem (CLT)

Thm:. If X_1, \ldots, X_n is a random sample with mean μ and variance σ^2 , then as $n \to \infty$, the limiting distribution of $\sqrt{n}(\bar{X}_n - \mu)/\sigma$ is standard normal, written as

$$\frac{\sqrt{n}(\bar{X}_n-\mu)}{\sigma}\to_d N(0,1).$$

Comments: converge in distribution as converge in cdf.

1
$$\lim_{n\to\infty} P(\sqrt{n}(\bar{X}_n - \mu)/\sigma \le z) \to \Phi(z)$$
 for any $z \in \Re$;
2 $\lim_{n\to\infty} P((T_o - n\mu)/(\sqrt{n}\sigma) \le z) \to \Phi(z)$ for any $z \in \Re$;

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Central Limit Theorem - CLT

A random sample $X_1, X_2, ..., X_n$ from ANY distribution. The sample mean $\overline{X} \ \overline{X}$ is asymptotically normally distributed.

[Think:] Asymptotically means when the sample size n is large. But how large is large?

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Central Limit Theorem - CLT

A random sample $X_1, X_2, ..., X_n$ from ANY distribution. The sample mean $\overline{X} \ \overline{X}$ is asymptotically normally distributed.

[Think:] Asymptotically means when the sample size n is large. But how large is large?

[Rule of Thumb :] If n > 30 the Central Limit Theorem can be used.

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Example	25			

Example 6.8 page 294

[e.g. 6.8] When a batch of a certain chemical product is prepared, the amount of a particular impurity in the batch is a random variable with mean 4.0 g and standard deviation 1.5 g. If 50 batches are independently prepared, what is the (approximate) probability that the sample average amount of impurity \bar{X} is between 3.5g and 3.8g ?

Outline General Properties of Sample Me	an For Normal Population (Exactly)	For general populations (asymptotically)	Но
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Other Applications of CLT (1)

Justify normal approximation of Binomial Distribution (we did this in Stat 318 but we didn't justify it using CLT)

Let $X \sim \text{Binomial}(n, p)$, if $np \geq 10$ and $nq \geq 10$, then

$$P(X \le x) = \Phi(rac{x - np + 0.5}{\sqrt{npq}})$$

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Applicatio	ons			



[6.20] The first assignment in a statistical computing class involves running a short program. If past experience indicates that 40% of all students will make no programming errors. compute the approximate probability that in a class of 50 students, at leas 25 will make no errors.

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Applications

Other Applications of CLT (2)

Let $X_1, X_2, ..., X_n$ be random sample from a distribution for which only positive values are possible ($P(X_i > 0) = 1$). Then if *n* is sufficiently large, the product $Y = X_1 X_2 \cdot ... \cdot X_n$ has approximately a lognormal distribution.

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Homework for this session

Part of HW 2:

- Section 6.2 page 298 11, 12, 14
 - Exercises $11(E(\bar{X}), V(\bar{X}))$
 - Exercises 12 (distribution of \bar{X} for normal distn)
 - Exercises 14 (CLT)

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