Overview of Chapter 7

Chapter 7 : Point Estimation

1. 7.1 General Concepts and Criteria
2. 7.2 Methods of Point Estimation
3. 7.3 Sufficiency
4. 7.4 Information and Efficiency

In short, the goal of chapter 7 is to learn how to find and evaluate point estimators.

Chapter 7 - Lecture 1
General concepts and criteria

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Parameter versus Estimator

Parameter refers to a value that represents the whole population.

Examples:

1. Binary variable (Smoke or not, Female or Male) : Bernoulli(p);
2. Number of customers: Poisson (λ);
3. Waiting time for a bus: exponential (λ);
4. Many other cases: height, $N(\mu, \sigma^2)$;

Parameter versus Estimator

However, parameters are usually unknown. We need to estimate them ! (How?)

Examples:

1. Bernoulli(p): $\hat{p}$ = sample proportion;
2. $N(\mu, \sigma^2)$: $\bar{X}_n$ and S^2;

Definitions

- A **point estimate** of parameter θ is a single number that can be regarded as a sensible value for θ.
- A point estimate is obtained by selecting a suitable statistic and computing its value from the given data. The selected statistic is called the **point estimator** of θ.
- Both the point estimate and point estimator of a parameter θ are denoted with $\hat{\theta}$.

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1. Estimator: random variables, before you see the data;
2. Estimate: numbers, after you see the data;

Example 1. If we are interested in the proportion of left-eye flounders p.We get a random sample $X_1, \ldots, X_{10}$. Let $X = 1$ if the fish is left-eye and $X = 0$ if the fish is not left-eye. Please give a point estimator of p.

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- point estimate: $\hat{p} = \frac{\sum_{i=1}^{10} x_i}{10} = 0.4$

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- point estimate: $\hat{\mu} = \frac{\sum_{i=1}^{5} x_i}{5} = 0.9168$

Common Criteria includes:

1. Unbiasedness;
2. Minimum Variance among unbiased estimators;
3. Minimum Mean Square Error (MSE);

Principle of Unbiased Estimation

bias of the point estimator $\hat{\theta}$:

$$\mathrm{E}\left(\hat{\theta}\right) - \theta$$

A point estimator $\hat{\theta}$ is said to be an **unbiased estimator** of θ if:

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Principle of Unbiased Estimation: To select those unbiased estimators.

Example 1. $\bar{X}$ is an unbiased estimator of μ and S^2 is an unbiased estimator of σ^2, under any distribution. (Example 7.6 for both editions)

1. You already did the proof for Hw2 #50
2. How about for general distribution?

Proposition

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If the distribution is **continuous and symmetric**, *then the median and the trimmed mean are also unbiased estimators of* μ.

Comments: The above shows that unbiased estimators are not unique! If we have several unbiased estimators, which one is better?

Principle of Minimum Variance Unbiased Estimator (MVUE)

Among all estimators $\hat{\theta}$ that are unbiased we choose the one with the minimum variance. This estimator is called **minimum variance unbiased estimator (MVUE)** of θ.

Example of MVUE

Theorem

If $X_1, \ldots, X_n$ is a random sample from a normal distribution with parameters μ and σ^2 $\left(X_i \sim N(\mu, \sigma^2)\right)$. Then the estimator $\hat{\mu} = \bar{X}$ is the MVUE for μ.

Example 7.7 both editions.

Given $X_1, \ldots, X_n$ is random sample from uniform $U(0, \theta)$. In order to estimate θ, there are two sensible estimators:

1. $\hat{\theta}_1 = 2\bar{X}$
2. $\hat{\theta}_2 = \frac{n+1}{n} \max\{X_1, \ldots, X_n\}$

It can be shown that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators for θ. However,

$$V(\hat{\theta}_1) = \frac{\theta^2}{3n} \qquad V(\hat{\theta}_2) = \frac{\theta^2}{n(n+2)}$$

As long as sample size $n > 1$, $V(\hat{\theta}_1) > V(\hat{\theta}_2)$, hence by MVUE, $\hat{\theta}_2$ is preferable.

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Mean Square Error $MSE(\hat{\theta}) = V(\hat{\theta}) + Bias(\hat{\theta})^2 = E(\hat{\theta} - \theta)^2$.

- MSE of an estimator is small if and only if both the variance and the bias is small.
- Hence the Principle of Minimum MSE is a trade off between bias and accuracy.
- When $\hat{\theta}$ is unbiased, MSE is the same as variance.

Homework for this session

HW 3: Due Feb 1

- Reading: Book session (1st) P326-335 ; (2nd) P332-342 (before More complications)
- Review problems in Hw1 and Qz1.
- Hand-in:
  - (1st) Page 342 8,10 ; (2nd) Page 348 8, 10
  - Redo a self-selected question which you did not get full score in Hw1. If you have full score in Hw1, you can skip this.
  - Given Normal distribution $N(\mu, 1)$, we want to estimate the μ. Suppose we get the random sample denoted $X_1, \ldots, X_n$. Both X_1 and $\bar{X}$ are sensible estimators for μ. Which one is preferable? What criterion do you use?