Hypotheses

1. Single population - One sample tests (Ch9)
   - Whether the mean of life-time length of human is 80 years;
2. Two populations - Two sample tests (Ch10)
   - Whether the new drug is more effective than the classical treatment.
3. Multiple populations - ANOVA (Ch11)
   - Are the four flavors of the ice-cream equally popular ? (Death By Chocolate, Peachy Paterno, Vanilla, Coffee Mocha Fudge)

1. For Normal population $N(\mu, 0.5^2)$, if you have two options : $\mu = -5$ and $\mu = 5$. The data you collected: $-4.5, -5.5, -5.1, -3.9, -6.1, -6.5, -5.3, -4.9, -4.7, -5.1$. Which value of μ will you choose?

2. However, if the data you collected were : $4.5, 5.1, 5.3, 4.9, 6.1, 5.5, 4.6, 4.7, 5.7, 6.3$ and you want to know whether $\mu > 5$.

Flu-X™

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Clinically proven to kill Influenza A and B viruses upon contact, prevent virus replication and contraction, reduce the severity of Colds and the Flu, and accelerate recovery.*

The result of over three years of extensive scientific and clinical research and development, *Flu-X*™ is a highly effective, natural herbal, oral anti-flu spray and is a patented scientific formulation of five standardized Chinese herb extracts.

Originally developed as an anti-SARS virus drug during China's SARS epidemic in 2003, *Flu-X*™ was used to treat SARS-infected patients and prevent SARS infection at several hospitals in China; due to its strong, broad anti-viral properties, it evolved into an anti-flu/anti-common cold product for the general public.

Read our consumer information here.
Professionals: Detailed clinical trials, scientific and technical information is here.

Video: Clinical Trials - from Company through FDA to Patient

Rhizoma Coptidis

World Health Organization

text title chapter title keyword

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WHO Home

WHO Health Systems and Services

WHO Medicines

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:: Themes & Keywords

WHO Monographs on Selected Medicinal Plants - Volume 1
(1999; 295 pages)

Rhizoma Coptidis

Definition

Rhizoma Coptidis is the dried rhizome of *Coptis chinensis* Franch, *Coptis deltoides* C.Y. Cheng et Hsiao, *Coptis japonica* Makino (Ranunculaceae), or other berberine-containing species of the same genus (*1*, *2*).

Medicinal uses

Uses supported by clinical data

None.

Uses described in folk medicine, not supported by experimental or clinical data

Treatment of arthritis, burns, diabetes, dysmenorrhoea, toothache, malaria, gout, and renal disease (*13*).

Out starting point is when you have the data, however, there are lots of stories about how to design the experiments to collect data. Sometimes, they're even harder than performing the tests itself.

In these three chapter, imagine that you would like to perform some testing procedures. The best motivation is when you want to need it!

Some extra for these chapters, you should be able to read software outputs for various tests.

Recommend readings ....

One sample z-test output from R

```
> x <- rnorm(25, 100, 5)
> z.test(x, 99, 5)

 One Sample z-test

data:  x
z = 0.9925, n = 25, Std. Dev. = 5, Std. Dev. of
the sample mean = 1, p-value = 0.3209
alternative hypothesis: true mean is not equal to 99
95 percent confidence interval:
  98.03258 101.95251
sample estimates:
mean of x
 99.99255
```

Ch9: Tests of Hypotheses Based on a single sample

9.1 Hypotheses and Test Procedures

9.2 Tests About a Population Mean

- Let n denote the sample size.

	σ known	σ unknown
Normal	n does not matter	under n is small
General	under n is large	

9.3 Tests Concerning a Population Proportion

- Large sample tests and small sample tests

9.4 P-values

Chapter 9 - Lecture 1
Hypotheses and Test Procedures

Yuan Huang

February 22, 2013

Elementary Procedure:

1. A hypothesis (assumption, statement) is given;
2. A testing procedure will be conducted to reject / fail to reject the hypothesis;

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1. A hypothesis (assumption, statement) is given;
2. A testing procedure will be conducted to reject / fail to reject the hypothesis;

As you may expect,

1. the hypothesis is about the population;
2. the testing procedure is based on the sample;

So nothing is deterministic !

Statistical Hypothesis

Statistical Hypothesis is a claim about the value of a parameter or about the value of several parameters, or about the entire probability distribution. There are two contradicting hypotheses:

- The **null hypothesis**
- The **alternative hypothesis**

Statistical Hypothesis

- The **null hypothesis**, represented by H_0, is a statement that there is nothing happening. The specific null hypothesis varies from problem to problem, but generally it can be thought of as the status quo, or no relationship, or no difference. In most situations, the researcher hopes to disprove or reject the null hypothesis.
- The **alternative hypothesis**, represented by H_1 or H_a, is a statement that something is happening. In most situations, this hypothesis is what the researcher hopes to prove. It may be a statement that the assumed status quo is false, or that there is a relationship, or that there is a difference.

Mind on Statistics, 3rd, P497

Step 1: State the hypotheses

- Identify the parameter of interest
- State the H_0
- State the H_1

Rules:

- H_0
  - Status quo, or no relationship, or no difference.
  - In most situations, the researcher hopes to disprove or reject the null hypothesis.
  - "$=$" always goes with H_0
- H_1
  - The assumed status quo is false, or that there is a relationship, or that there is a difference.
  - In most situations, this hypothesis is what the researcher hopes to prove.
  - never use "$=$" or "$\leq$", "$\geq$"
- H_0 and H_1 should be contradicting.

Example

The drying time of a type of paint under specified test conditions is known to be normally distributed with mean value 75 min and standard deviation 9 min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed with $\sigma = 9$.

Example

The drying time of a type of paint under specified test conditions is known to be normally distributed with mean value 75 min and standard deviation 9 min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed with $\sigma = 9$.

- $H_0 : \mu = 75$
- $H_a : \mu < 75$ (one-sided test)

Step 2: Select test statistics $T(X_1, \ldots, X_n)$

Definition

Test statistic is the function of sample data on which the decision whether to reject or not the null hypothesis will be based

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Definition

Test statistic is the function of sample data on which the decision whether to reject or not the null hypothesis will be based

Important: When you select a test statistic, you should be able to determine its sampling distribution under the null hypothesis.

Example(cont.)
Since we are interested in μ, it's very natural that our test statistics involve with $\bar{X}$.

Step3 : Get its corresponding null distribution for T. That is the distribution of T under H_0

Example(cont.)

In our example, we already know that the data is normally distributed with standard deviation 9. Hence, under $H_0 : \mu = 75$, we can completely determine the distribution that $X_i \sim N(\mu = 75, \sigma = 9)$.

Step3 : Get its corresponding null distribution for T. That is the distribution of T under H_0

Example(cont.)

In our example, we already know that the data is normally distributed with standard deviation 9. Hence, under $H_0 : \mu = 75$, we can completely determine the distribution that $X_i \sim N(\mu = 75, \sigma = 9)$.

Hence, under the $H_0 : \mu = 75$, we have $\bar{X} \sim N(75, \sigma = 9/\sqrt{n})$. Assume, out sample size $n = 25$, then $\sigma = 9/\sqrt{25} = 1.8$

Step 4: Determine the rejection/critical region C (or calculate the p-value)

Definition

Rejection region is a set of values of the test statistic for which the null hypothesis will be rejected.

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Example(cont.) In our example, a reasonable rejection region has the form $\bar{x} \leq c$, where the cutoff value c is suitably chosen.

To see how to choose a critical value, we have to introduce two types of errors first.

Type I and Type II error

Type I error $=P($ **Reject** $H_0|H_0$ **is true**$)$

- Probability of rejecting the null hypothesis when it is true
- It is also called the **size** of the test
- It is denoted with α

Type II error $=P($**Fail to reject** $H_0|H_0$ **is false**$)$

- Probability of not rejecting the null hypothesis when it is false
- It is denoted by β.

Power = 1 - Type II error

		True State of H_0	
		H_0 is True	H_0 is False
Decision	Fail to Reject H_0	Correct	Type II Error (β)
	Reject H_0	Type I Error (α)	Correct

Proposition

Suppose an experiment and a sample size are fixed and a test statistic is chosen. Then decreasing the size of the rejection region to obtain a smaller value of α results in a larger value of β for any particular parameter value consistent with H_α.

Therefore, This proposition says that once the test statistic and n are fixed, there is no rejection region that will simultaneously make both α and β small. A region must be chosen to effect a compromise between α and β.

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Because of the suggested guidelines for specifying H_0 and H_a, a type I error is usually more serious than a type II error. Therefore, we specify the largest value of α that can be tolerated and find a rejection region accordingly.

Therefore, This proposition says that once the test statistic and n are fixed, there is no rejection region that will simultaneously make both α and β small. A region must be chosen to effect a compromise between α and β.

Because of the suggested guidelines for specifying H_0 and H_a, a type I error is usually more serious than a type II error. Therefore, we specify the largest value of α that can be tolerated and find a rejection region accordingly.

Usually, α is chosen to be 0.1, 0.05 and 0.01.

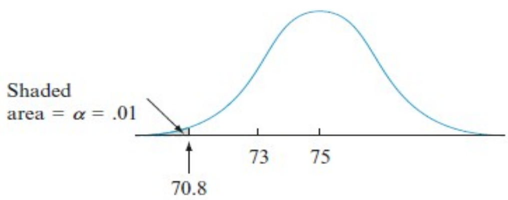
For our examples, take $\alpha = 0.05$.
Let's see two examples:

1. Given $c = 70.8$, calculate the type I error and type II error.
2. Obtain c to make $\alpha = 0.05$.

1 Given $c = 70.8$, calculate the type I error and type II error.

- **Type I error** α should be calculate given $H_0 : \mu = 75$ is true. Hence, by Step 3, we have $\bar{X} \sim N(75, \sigma = 1.8)$.

$$\begin{aligned}
\alpha &= P(\text{type I error}) = P(H_0 \text{ is rejected when it is true}) \\
&= P(\overline{X} \leq 70.8 \text{ when } \overline{X} \sim \text{normal with } \mu_{\overline{X}} = 75, \sigma_{\overline{X}} = 1.8) \\
&= \Phi\left(\frac{70.8 - 75}{1.8}\right) = \Phi(-2.33) = .01
\end{aligned}$$

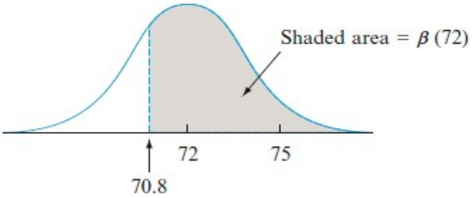


Modern Mathematical Statistics with Applications, 2rd, P431

- **Type II error** β should be calculate alternative μ.

(a) If true $\mu = 72$,

$$\begin{aligned}\beta(72) &= P(\text{type II error when } \mu = 72) \\ &= P(H_0 \text{ is not rejected when it is false because } \mu = 72) \\ &= P(\overline{X} > 70.8 \text{ when } \overline{X} \sim \text{normal with } \mu_{\overline{X}} = 72, \sigma_{\overline{X}} = 1.8) \\ &= 1 - \Phi\left(\frac{70.8 - 72}{1.8}\right) = 1 - \Phi(-.67) = 1 - .2514 = .7486\end{aligned}$$

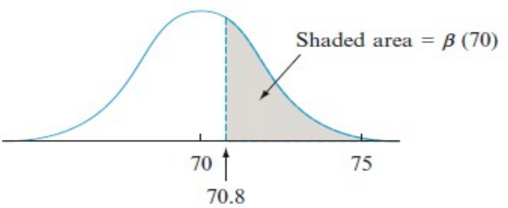


Modern Mathematical Statistics with Applications, 2rd, P431

- **Type II error** β should be calculate alternative μ.

(b) If true $\mu = 70$,

$$\begin{aligned}\beta(70) &= P(\text{type II error when } \mu = 70) \\ &= P(H_0 \text{ is not rejected when it is false because } \mu = 70) \\ &= P(\overline{X} > 70.8 \text{ when } \overline{X} \sim \text{normal with } \mu_{\overline{X}} = 70, \sigma_{\overline{X}} = 1.8) \\ &= 1 - \Phi\left(\frac{70.8 - 70}{1.8}\right) = 1 - \Phi(.444) = .3300\end{aligned}$$



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2 Obtain c to make $\alpha = 0.05$.

$$\begin{aligned} \alpha &= P(\text{make type I error}) = P(\text{ Reject } H_0 | H_0 \text{ is true}) \\ &= P(\bar{X} < c | \bar{X} \sim N(75, \sigma = 1.8)) \\ &= P(\frac{\bar{X} - 75}{1.8} < \frac{c - 75}{1.8}) = P(Z < \frac{c - 75}{1.8}) = \alpha = 0.05 \end{aligned}$$

Hence, $\frac{c-75}{1.8} = z_{0.95} = -1.645 \rightarrow c = 72.039$

Step 5: Make a decision

Recall: in our example,

- $H_0 : \mu = 75$
- $H_a : \mu < 75$ (one-sided test)

1. If the sample value of T falls in reject region C, we reject H_0. You can state :
   - "There is evidence / The data supports that $\mu < 75$ at the significance level α"
2. Otherwise we fail to reject H_0. You can state:
   - "There is not adequate support to the conclusion at the significance level α "
   - "There is not enough evidence to reject the null that $\mu = 75$. at the significance level α "

Summary: The Procedure of Hypotheses Testing

In the hw or exams, Please specify the five steps explicitly.

Step 1: State the hypotheses

Step 2: Select test statistics $T(X_1, \ldots, X_n)$

Step 3: Get its corresponding null distribution for T. That is the distribution of T under H_0

Step 4: Determine the rejection/critical region C (or calculate the p-value)

Step 5: Make a decision : If the sample value of T does fall in reject region C, we reject H_0; otherwise we fail to reject H_0.

Summary of our example

[Example 9.2] The drying time of a type of paint under specified test conditions is known to be normally distributed with mean value 75 min and standard deviation 9 min. Chemists have proposed a new additive designed to decrease average drying time. It is believed that drying times with this additive will remain normally distributed with $\sigma = 9$.

Step 1: State the hypotheses:

- $H_0 : \mu = 75$
- $H_a : \mu < 75$ (one-sided test)

Step 2: Select test statistics $T(X_1, \ldots, X_n)$

- $\bar{X}$

Step 3: Get its corresponding null distribution for T. That is the distribution of T under H_0

- under the $H_0 : \mu = 75$, we have $\bar{X} \sim N(75, \sigma = 9/\sqrt{n})$. Assume, out sample size $n = 25$, then $\sigma = 9/\sqrt{25} = 1.8$

Summary of our example (cont.)

Step 4: Determine the rejection/critical region C

- first fix Type I error $\alpha = 0.05$

$$\begin{aligned}\alpha &= PP(\text{ Reject } H_0 | H_0 \text{ is true}) = P(\bar{X} < c | \bar{X} \sim N(75, \sigma = 1.8)) \\ &= P(\frac{\bar{X} - 75}{1.8} < \frac{c - 75}{1.8}) = P(Z < \frac{c - 75}{1.8}) = \alpha = 0.05\end{aligned}$$

Hence, $\frac{c-75}{1.8} = z_{0.95} = -1.645 \rightarrow c = 72.039$. Until now, we can write down our rejection region $\{\bar{x} < 72.039\}$.

Step 5: Make a decision : If the sample value of T falls in reject region C, we reject H_0; otherwise we fail to reject H_0.

- If $\bar{x} = 70$, then it falls in the rejection region. There is evidence that $\mu < 75$ at the significance level 0.05.
- If $\bar{x} = 79$, then it does not fall in the rejection region. There is not enough evidence to reject the null that $\mu = 75$ at the significance level α.