Math/Stat319 for chapter 10

Here listed the rejection region method. You also need to know how to apply the p-value method.

1. Two independent normal population with known σs. (exact)

   - $100(1-\alpha)\%$ Confidence Interval of $\mu_1 - \mu_2$ is

   $$(\bar{x}_m - \bar{y}_n - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}, \bar{x}_m - \bar{y}_n + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}})$$

   - Null hypothesis: $H_o : \mu_1 - \mu_2 = c_0$
   - Test statistics : $Z = \frac{\bar{X}_m - \bar{Y}_n - c_0}{\sqrt{\sigma_1^2/m + \sigma_2^2/n}} \sim N(0,1)$
   - Rejection Region:
     (a) For $H_1 : \mu_1 - \mu_2 > c_0$, we reject H_o when $z > z_\alpha$;
     (b) For $H_1 : \mu_1 - \mu_2 < c_0$, we reject H_o when $z < -z_\alpha$;
     (c) For $H_1 : \mu_1 - \mu_2 \neq c_0$, we reject H_o when $|z| > z_{\alpha/2}$.

2. Two independent population with unknown σs. (approximate)

   - extra condition for CI: $m > 40$, $n > 40$
   - extra condition for testing: $m > 40$, $n > 40$
   - $100(1-\alpha)\%$ Confidence Interval of $\mu_1 - \mu_2$ is :

   $$(\bar{x}_m - \bar{y}_n - z_{\alpha/2}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}, \bar{x}_m - \bar{y}_n + z_{\alpha/2}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}).$$

   - Null hypothesis: $H_o : \mu_1 - \mu_2 = c_0$
   - Test statistics : $Z = \frac{\bar{X}_m - \bar{Y}_n - c_0}{\sqrt{S_1^2/m + S_2^2/n}} \sim N(0,1)$
   - Rejection Region:
     (a) For $H_1 : \mu_1 - \mu_2 > c_0$, we reject H_o when $z > z_\alpha$;
     (b) For $H_1 : \mu_1 - \mu_2 < c_0$, we reject H_o when $z < -z_\alpha$;
     (c) For $H_1 : \mu_1 - \mu_2 \neq c_0$, we reject H_o when $|z| > z_{\alpha/2}$.

3. Two independent normal population with unknown σs. (exact)

   (a) Unpooled case. Use $\nu = \dfrac{\left(\frac{s_1^2}{m} + \frac{s_2^2}{n}\right)^2}{\frac{\left(\frac{s_1^2}{m}\right)^2}{m-1} + \frac{\left(\frac{s_2^2}{n}\right)^2}{n-1}}$

   - $100(1-\alpha)\%$ Confidence Interval of $\mu_1 - \mu_2$ is :

   $$(\bar{x} - \bar{y} - t_{\nu,\frac{\alpha}{2}}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}, \bar{x} - \bar{y} + t_{\nu,\frac{\alpha}{2}}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}})$$

- Test statistic :

$$T = \frac{\bar{X} - \bar{Y} - c_0}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}} \sim t_\nu$$

- Rejection Regions:
  - If $H_A : \mu_1 - \mu_2 > c_0$, $t \geq t_{\nu,\alpha}$
  - If $H_A : \mu_1 - \mu_2 < c_0$, $t \leq -t_{\nu,\alpha}$
  - If $H_A : \mu_1 - \mu_2 \neq c_0$, $t \leq -t_{\nu,\alpha/2}$ and $t \geq t_{\nu,\alpha/2}$

(b) Pooled case. Make assumption that $\sigma_1 = \sigma_2$. Use $S_p^2 = \dfrac{(m-1)S_1^2 + (n-1)S_2^2}{n+m-2}$

- $100(1-\alpha)\%$ Confidence Interval of $\mu_1 - \mu_2$ is :

$$(\bar{x} - \bar{y} - t_{\alpha/2,m+n-2}\sqrt{s_p^2(\frac{1}{m} + \frac{1}{n})}, \bar{x} - \bar{y} + t_{\alpha/2,m+n-2}\sqrt{s_p^2(\frac{1}{m} + \frac{1}{n})}).$$

- Test statistic :

$$T = \frac{\bar{X} - \bar{Y} - (c_0)}{\sqrt{S_p^2\left(\frac{1}{m} + \frac{1}{n}\right)}} \sim t_{m+n-2}$$

- Rejection Regions:
  i. If $H_A : \mu_1 - \mu_2 > c_0$, $t \geq t_{m+n-2,\alpha}$
  ii. If $H_A : \mu_1 - \mu_2 < c_0$, $t \leq -t_{m+n-2,\alpha}$
  iii. If $H_A : \mu_1 - \mu_2 \neq c_0$, $t \leq -t_{m+n-2,\alpha/2}$ and $t \geq t_{m+n-2,\alpha/2}$

4. Two independent population proportions. (approximate)

- extra condition for CI: $m\hat{p}_1 > 10, m(1-\hat{p}_1) > 10$, $n\hat{p}_2 > 10, n(1-\hat{p}_2) > 10$
- extra condition for testing: $m\hat{p} \geq 10, m(1-\hat{p}) \geq 10, n\hat{p} \geq 10, n(1-\hat{p}) \geq 10$, where $\hat{p} = \dfrac{x+y}{m+n} = \dfrac{m}{m+n}\hat{p}_1 + \dfrac{n}{m+n}\hat{p}_2$
- $100(1-\alpha)\%$ Confidence Interval of $\mu_1 - \mu_2$ is :

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{1}{m}\hat{p}_1(1-\hat{p}_1) + \frac{1}{n}\hat{p}_2(1-\hat{p}_2)}$$

- Null hypothesis: $H_0 : p_1 = p_2$
- Test statistics : $Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{m} + \frac{1}{n}\right)}} \sim N(0,1)$
- Rejection Region:
  (a) For $H_1 : p_1 - p_2 > 0$, we reject H_o when $z > z_\alpha$;
  (b) For $H_1 : p_1 - p_2 < 0$, we reject H_o when $z < -z_\alpha$;
  (c) For $H_1 : p_1 - p_2 \neq 0$, we reject H_o when $|z| > z_{\alpha/2}$.

5. Paired data analysis → Take different between the paired data and perform the corresponding one sample analysis as in chapter 8 and 9.